Estimation Of Linearised Fluid Film Coefficients In A Rotor Bearing System Subjected To Random Excitation

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ABSTRACT

An identification procedure for estimating system parameters of rotating machines specially the bearing fluid film coefficients is described. A rigid rotor running on fluid film bearings is excited by random force. An approach for parameter estimation is developed through a first order Volterra kernel representation of the system response. A two-degree-of-freedom system with cross coupling effects is considered in the present study. In addition to direct FRFs (frequency response functions) or first order direct kernels, such a system requires definition of cross FRFs or cross-kernels and their estimation. The expressions for direct and cross-kernels are constructed in frequency domain. The parameter estimation procedure involves extraction of first order Volterra kernels from the response of the system to a Gaussian white noise excitation. Eight linearised fluid film coefficients are estimated by fitting the measured complex frequency response function to those theoretically calculated. The procedure is illustrated through numerical simulation. The assumptions involved and the approximations are discussed. The influence of excitation force and probable measurement noise on estimates is illustrated through numerical simulation.

INTRODUCTION

Rotor bearing systems play a vital role in various fields of engineering. In rotating machinery, the dynamic stiffness of the structure that supports the rotating shaft has considerable effect on the machine vibration. It not only affects the critical speed of the machine, the vibration in between the critical speeds but also affects the forces transmitted from the machine to the ground and noise emanating from the machine. Since the elements of support structure are assembled in series so the most flexible element has major effect on machine vibration characteristic. In large rotating machines bearing is the most flexible element and this is frequently a fluid film bearing.

Bearing characteristic has a major influence on the performance of the rotor bearing system. It has been found that rotor-journal-bearing system is a tribo-dynamic system and the dynamic characteristic of fluid film and other tribo elements are time varying [1]. The impedance of most fluid film bearings can be represented with reasonable accuracy with eight linear springs and damping coefficients as described by Lund [6]. Over the past few years the principal of structural identification has been applied in different time domain
and frequency-domain techniques of fluid film coefficients estimation [2,3,4]. Each method has some relative merits and demerits. The work presented in reference [2] showed that a frequency domain approach was less susceptible to noise and offer more convenience for being implemented on line. An identification procedure for estimating bearing film coefficients is suggested in order to reduce the influence of phase measurements errors on estimated results [1]. An experimental procedure for measuring bearing impedance using multi frequency test signals give more reliable data and agree with the theoretical prediction within 20% [5].

When the excitation is due to mass unbalance it is single frequency excitation and the response is large in the vicinity of critical speed. However, when the excitation is from outside source such as in case of support excitation can be a single frequency type or random involving several frequencies. Lund [6] carried out response spectral density analysis of rotor systems due to stationary random excitation of the base considering excitation in vertical direction only. Bhat [7] used modal analysis to analyze the response of an undamped single mass rotor mounted on fluid film bearings. Tessarzik et al. [8] analyzed experimentally the response of turbo rotor to random excitation.

The aim of the work described in this paper is to develop a method for estimation of the parameters. Two degree of freedom system with direct as well as cross-coupled stiffness and damping coefficients is considered. An approach for parameter estimation is developed through a first order Volterra kernel representation of the system response [9]. Using frequency domain analysis the first order kernel is extracted from the measurement of applied force and response. The influence of excitation force, measurement noise and errors on the estimated results is analyzed. Complex curve fitting technique is employed to improve the accuracy of estimated coefficients. The procedure is illustrated through numerical simulation. In most conditions of laminar flow the inertia coefficients are neglected and therefore we are considering only the eight coefficients of the fluid film bearing.

**SYSTEM MODEL AND GOVERNING EQUATIONS OF MOTION**

A rigid rotor supported on fluid film bearing can be represented as shown in Fig.1, where the fluid film is represented by eight spring & damping coefficients as discussed earlier. The equation of motion in the vertical and horizontal direction relating the displacement to the forces applied to it is given by

\[
\begin{align*}
mx'' + c_{xx}x + c_{yx}y + k_{xx}x + k_{yx}y &= f_1(t) \\
m_y'' + c_{yy}y + c_{yx}x + k_{yy}y + k_{xy}x &= f_2(t)
\end{align*}
\]

where \( m \) is the mass, \( c_{xx}, c_{yy} \) are direct linear damping terms; \( c_{yx}, c_{xy} \) are cross-coupled linear damping coefficients and \( k_{xx}, k_{yy} \) are the direct linear stiffness terms, while \( k_{xy}, k_{yx} \) are the cross-coupled linear stiffness term. \( f_1(t), f_2(t) \), in the above equation represents the excitation given to the system in \( x \) \& \( y \) direction.
In order to write the equation of motion in non-dimensional form. Let us define

\[ \tau = \omega_n t, \quad \omega_n = \sqrt{\frac{k_{xx}}{m}}, \quad \xi_{ii} = \frac{c_{ii} \omega_n}{2m \omega_n^2}, \quad \eta = \frac{x}{X_{st}}, \]

\[ \gamma = \frac{y}{X_{st}}, \quad \lambda_{ij} = \frac{k_{ij}}{k_{xx}}, \quad \bar{f}_i(\tau) = \frac{f_i(\tau)}{F_{\text{max}}}, \quad \text{and} \quad X_{st} = \frac{F_{\text{max}}}{k_{xx}} \]

where, \( i = x, y \); \quad \text{and} \quad j = x, y

Therefore the equations of motion can be written in a non-dimensional form as

\[ ^t \eta'' + 2 \xi_{xx} ^t \eta' + 2 \xi_{xy} ^t \eta' + \gamma \eta + \lambda_{xx} ^t \eta = \bar{f}_1(\tau) \]

\[ ^t \xi_{xy} ^t \eta' + 2 \xi_{xy} ^t \eta' + \lambda_{xy} ^t \eta + \lambda_{xx} ^t \eta = \bar{f}_2(\tau) \]

where \( (\cdot) \) denotes differentiation with respect to \( \tau \).

The solution of equations (4) and (5) is represented in terms of Volterra operators as

\[ ^t \eta(\tau) = ^t T \bar{f}_1(\tau), \bar{f}_2(\tau) \]

\[ ^t \eta(\tau) = ^t T \bar{f}_1(\tau), \bar{f}_2(\tau) \]

Considering the first order Volterra kernel representation of response, we can write

\[ ^t \eta^{(i)}(\tau) = ^t H^{(i)} \bar{f}_i(\tau) \]

where

\[ ^t H^{(i)} \bar{f}_i(\tau) = \int_{-\infty}^{\infty} h^{(i)}(\tau - \tau_i) d\tau_i \]

with \( \kappa \) denotes \( x \) or \( y \) for \( i=1,2 \)

Taking Laplace transform of equation (4) & (5) we get
\[ s^2 \hat{\eta}(s) + 2s \xi_{xx} \hat{\eta}(s) + 2s \xi_{xy} \hat{\eta}(s) + \xi_{yy} \hat{\eta}(s) + \lambda_{xy} \hat{\eta}(s) = F_1(s) \]  \hspace{1cm} (10)

\[ s^2 \hat{\eta}(s) + 2s \xi_{yy} \hat{\eta}(s) + 2s \xi_{yx} \hat{\eta}(s) + \lambda_{yy} \hat{\eta}(s) + \lambda_{yx} \hat{\eta}(s) = F_2(s) \]  \hspace{1cm} (11)

Solving the above two simultaneous equations (10) and (11), the solutions for the operators \( \hat{\eta}(s) \) and \( \hat{\eta}(s) \) are

\[ \hat{\eta}(s) = \frac{(s^2 + 2\xi_{yy}s + \lambda_{yy})F_1(s) - (2\xi_{yx}s + \lambda_{xy})F_2(s)}{(s^2 + 2\xi_{xx}s + 1)(s^2 + 2\xi_{yy}s + \lambda_{yy}) - (2\xi_{yx}s + \lambda_{xy})(2\xi_{xy}s + \lambda_{xy})} \]  \hspace{1cm} (12)

\[ \hat{\eta}(s) = \frac{(s^2 + 2\xi_{xy}s + 1)F_2(s) - (2\xi_{yx}s + \lambda_{xy})F_1(s)}{(s^2 + 2\xi_{xx}s + 1)(s^2 + 2\xi_{yy}s + \lambda_{yy}) - (2\xi_{yx}s + \lambda_{xy})(2\xi_{xy}s + \lambda_{xy})} \]  \hspace{1cm} (13)

which give

\[ \hat{H}_{1}^{(1)}(s) = \frac{(s^2 + 2\xi_{yy}s + \lambda_{yy})}{(s^2 + 2\xi_{xx}s + 1)(s^2 + 2\xi_{yy}s + \lambda_{yy}) - (2\xi_{yx}s + \lambda_{xy})(2\xi_{xy}s + \lambda_{xy})} \]  \hspace{1cm} (14)

\[ \hat{H}_{1}^{(2)}(s) = \frac{-(2\xi_{xy}s + \lambda_{xy})}{(s^2 + 2\xi_{xx}s + 1)(s^2 + 2\xi_{yy}s + \lambda_{yy}) - (2\xi_{yx}s + \lambda_{xy})(2\xi_{xy}s + \lambda_{xy})} \]  \hspace{1cm} (15)

and similarly

\[ \hat{H}_{1}^{(2)}(s) = \frac{-{(s^2 + 2\xi_{xy}s + 1)}}{(s^2 + 2\xi_{xx}s + 1)(s^2 + 2\xi_{yy}s + \lambda_{yy}) - (2\xi_{yx}s + \lambda_{xy})(2\xi_{xy}s + \lambda_{xy})} \]  \hspace{1cm} (16)

\[ \hat{H}_{1}^{(2)}(s) = \frac{(s^2 + 2\xi_{xx}s + 1)}{(s^2 + 2\xi_{xx}s + 1)(s^2 + 2\xi_{yy}s + \lambda_{yy}) - (2\xi_{yx}s + \lambda_{xy})(2\xi_{xy}s + \lambda_{xy})} \]  \hspace{1cm} (17)

Also

\[ \hat{H}_{1}^{(1)}(s) = \frac{(s^2 + 2\xi_{yy}s + \lambda_{yy})}{(2\xi_{yx}s + \lambda_{xy})} \]  \hspace{1cm} (18)

\[ \hat{H}_{1}^{(2)}(s) = \frac{(2\xi_{xy}s + \lambda_{xy})}{(s^2 + 2\xi_{xx}s + 1)} \]  \hspace{1cm} (19)

**COMPUTER SIMULATION**

The procedure is illustrated through numerical simulation of the response for the non-dimensional coupled equations (4) and (5). The forcing functions in the equations are normalized zero mean random forces, \( f_1(\tau) \) and \( f_2(\tau) \). The excitation forces are simulated through random number generating subroutines and are normalized with respect to their maximum values. A typical sample of such excitation is shown in Fig. 2. Owing to the statistical nature of the problem, the procedure is illustrated for various sets of direct and coupled stiffness and damping parameters. The following case studies (Table-1) have been designed to study the influence of direct and coupled parameters and errors involved.
Table 1: Case Studies

<table>
<thead>
<tr>
<th></th>
<th>$\xi_{xx}$</th>
<th>$\xi_{xy}$</th>
<th>$\xi_{yx}$</th>
<th>$\xi_{yy}$</th>
<th>$\lambda_{xx}$</th>
<th>$\lambda_{xy}$</th>
<th>$\lambda_{yx}$</th>
<th>$\lambda_{yy}$</th>
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<tbody>
<tr>
<td>Case-I</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>1.00</td>
<td>1.50</td>
<td>1.50</td>
<td>3.00</td>
</tr>
<tr>
<td>Case-II</td>
<td>0.10</td>
<td>0.01</td>
<td>0.01</td>
<td>0.10</td>
<td>1.00</td>
<td>-0.50</td>
<td>-0.50</td>
<td>1.00</td>
</tr>
<tr>
<td>Case-III</td>
<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
<td>0.05</td>
<td>1.00</td>
<td>0.50</td>
<td>0.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Case-IV</td>
<td>0.10</td>
<td>0.01</td>
<td>0.01</td>
<td>0.10</td>
<td>1.00</td>
<td>1.50</td>
<td>1.50</td>
<td>3.00</td>
</tr>
</tbody>
</table>

The response is computationally simulated, for various sets of numerical values of the parameters in equations (4) and (5). The governing equations are then numerically solved using a fourth order Runge-Kutta method, to obtain the responses in $x$ and $y$ directions. These responses are fed as inputs to the parameter estimation algorithm. The various direct and cross-coupled first order FRFs are extracted from the response and consequently parameter estimation is carried out. The estimated parameters are compared with those originally used for the simulation of the response and accuracy of the estimates and errors involved are studied.

RESULTS AND DISCUSSION

The power spectrum of the random force averaged over the ensemble of 2000 samples is shown in Fig. 3. The non-dimensional responses $\eta(\tau)$ and $\eta(\tau)$ have been numerically generated in the non-dimensional time range 0-2048. The ensemble is constructed from 2000 different samples of the simulated random force. The response spectra are fed as input to the parameter estimation algorithm. The first order direct and cross Volterra kernels i.e. $H^x_1(\omega)$ and $H^y_1(\omega)$, Fig 4 and Fig. 5 respectively are computed using equations (17) and (16) for case-I. These kernels exhibit peak responses corresponding to two critical frequencies. The eight coefficients i.e. $\lambda_{xx}$, $\lambda_{yy}$, $\lambda_{xy}$, $\lambda_{yx}$, $\xi_{xx}$, $\xi_{xy}$, $\xi_{yx}$, and $\xi_{yy}$ are obtained using complex curve fit technique [10]. The parameters thus estimated are

$$\begin{align*}
\lambda_{xx} &= 1.0000003245, \quad \lambda_{xy} = 1.4797083297, \quad \lambda_{yx} = 1.4924137005, \quad \lambda_{yy} = 2.9682544926 \\
\xi_{xx} &= 0.0107308061, \quad \xi_{xy} = 0.0105932223, \quad \xi_{yx} = 0.0104724361, \quad \xi_{yy} = 0.0104968073
\end{align*}$$

The exact values of the above coefficients are those used as input for numerical simulation of the response. While, the direct as well as cross stiffness terms are seen to be estimated with a very good degree of accuracy, the error in the damping estimates is higher. The curves of Figs. 6 and Fig. 7 shows the errors incurred in the estimate of $H^x_1(\omega)$ and $H^y_1(\omega)$ due to statistical nature of the Fast Fourier Transform computational procedure and the finite length of samples. The exact values of these
kernels are those obtained from expressions, after direct substitution of numerical values of the coefficients employed for simulation of the responses. The normalized random error as known, can be seen to be maximum in the vicinity of peak responses at critical frequencies. Since the numerical error is higher in the vicinity of the peaks, the error in damping estimates may be reduced by taking care of these zones. In cases II and III also the estimates are found to be satisfactory. Case IV has been studied to consider the effect of damping where the peaks are found to be no sharper as in Case-I, whereas the parameters are found to be within the acceptable limits.

CONCLUSIONS
The Volterra kernels extraction is proposed for the estimation of eight-linearised fluid film parameter in frequency domain. Complex curve fitting technique is employed in order to improve the accuracy of estimated coefficients. The effect of various types of disturbances have been analyzed using simulated data. The procedure developed gives very good engineering estimates of fluid film parameters. It has been found that the normalized random error is maximum in the vicinity of peak responses at critical frequencies; the error in damping estimates may be reduced by taking care of these zones. The simulated results also show the effect of ensemble size on estimated results. It has been found that large sample size is preferable for obtaining sharp peaks that gives more accurate results. It has been also found that the procedure developed is robust to the measurement noise, which can be usefully employed for design of experiments.

NOMENCLATURE

\( m \)  
\( \tau \)  
\( c_{xx}, c_{yy}, c_{xy}, c_{yx} \)  
\( f_1(t), f_2(t) \)  
\( \tilde{f}_1(t), \tilde{f}_2(t) \)  
\( F_{\text{max}} \)  
\( H_1^{(1)}(S), H_1^{(2)}(S) \)  
\( H_1^{(1)}(S), H_1^{(2)}(S) \)  
\( k_{xx}, k_{yy}, k_{xy}, k_{yx} \)  
\( \lambda_{xx}, \lambda_{yy}, \lambda_{xy}, \lambda_{yx} \)  
\( \xi_{xx}, \xi_{yy}, \xi_{xy}, \xi_{yx} \)  
\( \eta_x, \eta_y \)  
\( \omega_n \)  

mass  
nondimensional time  
direct and cross damping parameters  
input force in \( x \) and \( y \) directions  
normalized input force in \( x \) and \( y \) directions  
maximum value of the input force  
first order cross Volterra kernel transform  
first order direct Volterra kernel transform  
direct and cross stiffness parameters  
nondimensional direct and cross stiffness parameters  
nondimensional direct and cross damping parameters  
nondimensional response in \( x \) and \( y \) direction  
natural frequency
Fig. 2: Typical sample of normalised input force

Fig. 3: Power spectrum of input force

Fig. 4: First order direct Volterra kernel \( \mathcal{H}_1^{(2)}(\omega) \) for Case-I

Fig. 5: First order cross Volterra kernel \( \mathcal{H}_1^{(2)}(\omega) \) for Case-I

Fig. 6: Normalised Error in the estimate of \( \mathcal{H}_1^{(2)}(\omega) \) for Case-I

Fig. 7: Normalised Error in the estimate of \( \mathcal{H}_1^{(2)}(\omega) \) for Case-I
REFERENCES


