Stability And Unbalance Response Of Rotor Bearing System

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ABSTRACT

This paper presents stability and unbalance response prediction for symmetric rotor bearing system using simple but realistic models. The rotor system is described with kinetic energy and potential energy functionals that fully account for translational inertia, rotary inertia, and gyroscopic coupling. Rayleigh–Ritz method is used to describe the flexural displacements. The displacement function chosen is the first mode shape of a classical free–free beam in two orthogonal planes with constant cross section in bending. The rotor motion is derived using Lagrange’s equations, and expressed as state space form that preserves the skew symmetric gyroscopic matrix within the shaft and disks as well as cross coupling effects of journal bearings. Results are presented for threshold speed of instability, natural frequencies as a function of rotor speed (Campbell diagram), and unbalance response. Influence of variation of journal bearing parameters (bearing length, radial clearance, oil viscosity) and distance between the flexible bearing supports on the rotor threshold speed of instability is evaluated.

INTRODUCTION

A study of dynamic behavior of rotors mounted on fluid film bearings is very important with increase in demand of power requirements and decrease in weight of rotating machinery. The literature on rotor dynamics is quite extensive. Because of its fundamental importance in the design of rotating machinery, rotor dynamics will persist as an active research area in future also. Several well established methods are nowadays available for linear analysis of rotor bearing systems, such as: transfer matrix [1], finite elements [2], dynamic stiffness [3] and assumed modes [4]. Lallane and Farris [5] derived the equations of motion for simple rotor models to show the basic phenomena occurring in rotordynamics. They have analyzed natural frequencies as a function of
speed of rotation, instability and responses to forces of excitation for symmetric/asymmetric rotor.

The instability of rotors in fluid film bearings is generally attributed to the self excited vibrations known as oil whirl/whip of a rotor. Fluid induced self excited vibrations (fluid whirl and whip instability) in rotor bearing systems, theoretical modeling and experimental determination are presented in detail by Muszynska [6] for rotors supported on a rigid (brass oilite) bearing at the inboard end and 360° oil lubricated journal bearing at the outboard end. Based on Lund’s [7] linearized stability analysis, Rao [8] derived analytical expressions for Sommerfeld number, stiffness and damping coefficients as a function of steady state eccentricity ratio of a plain cylindrical journal bearing.

The present work focuses on stability and unbalance response prediction for symmetric rotor bearing system using assumed modes or Rayleigh–Ritz method, where the rotor displacement field is represented as superposition of basis functions of the space variable and generalized coordinates as functions of time. The expressions for kinetic and strain energies of the rotor elements and virtual work of fluid film bearings are calculated. Langrange’s equations are applied to describe the dynamics of rotor system. The Rayleigh–Ritz method can be accurate with a suitable choice of assumed modes and it has the advantage of representing the detail of a complex rotor system with relatively few degrees of freedom.

THEORY

Rotor-Bearing Model

Figure 1 [8] shows the coordinate system of reference for the simplified analysis of shaft with three disks supported by bearings at both ends. The support bearings are similar and described by classical 8 linearized spring and damping coefficients. To determine the gyroscopic inertial effects, the present analysis applies Stodola–Green model [9]. In a linearized theory, the shaft kinetic energy for the flexural displacements \( u \) and \( w \) relate to the orthogonal fixed transverse directions \( x \) and \( z \) respectively, and cross sectional rotations \( \theta \) and \( \psi \) which are Eulerian angles relative to those axes are:

\[
T_x = \frac{\rho S}{2} \int_0^L \left( \dot{u}^2 + \dot{w}^2 \right) \, dy + \frac{\rho I}{2} \int_0^L \left( \dot{\theta}^2 + \dot{\psi}^2 \right) \, dy + \rho I \int_0^L \left( \omega^2 + 2\omega \dot{\psi} \dot{\theta} \right) \, dy
\]  

(1)

The kinetic energy of each attached disk “d” are:

\[
T_d = \frac{1}{2} M_D \left( \dot{u}^2 + \dot{w}^2 \right) + \frac{1}{2} I_{Dx} \left( \dot{\theta}^2 + \dot{\psi}^2 \right) + \frac{1}{2} I_{Dy} \left( \omega^2 + 2\omega \dot{\psi} \dot{\theta} \right)
\]  

(2)

The strain energy of the shaft contains the influence of bending deformation only without taking into consideration of shear deformation [10] and it is assumed that coupling of torsional and axial loadings with the flexural motion is unimportant. The resulting potential energy functional is:
Bearing forces are taken to be arbitrary linear functions of the transverse displacements and velocity components at the bearing locations. The virtual work done by the forces at the bearing locations “b” is:

$$\delta W_b = -\left( K_{xu} u + K_{xz} w + B_{xu} \dot{u} + B_{xz} \dot{w} \right) \delta u$$

$$- \left( K_{zu} u + K_{zz} w + B_{zu} \dot{u} + B_{zz} \dot{w} \right) \delta w$$

Application of Langrange’s equations to the expressions for the kinetic energy of shaft – disk assembly with unbalance mass, strain energy of shaft and virtual work due to bearing reactions on the shaft leads to a set of coupled second order linear differential equations of the form:

$$m \ddot{q}_1 + B_{11} \dot{q}_1 + B_{12} \dot{q}_2 + K_1 q_1 + K_{12} q_2 = m_u f(l_1) d_u \alpha^2 \sin \omega t$$

$$m \ddot{q}_2 + B_{21} \dot{q}_1 + B_{22} \dot{q}_2 + K_2 q_1 + K_{22} q_2 = m_u f(l_2) d_u \alpha^2 \cos \omega t$$

where

$$m = \sum_d \left[ M_D f^2(l) + I_{Dx} g^2(l) \right] + \rho S \int_0^l f^2(y) dy + \rho I \int_0^l g^2(y) dy$$

$$M_D = \pi \left( R_2^2 - R_1^2 \right) h \rho$$

$$I_{Dx} = I_{Dc} = \frac{M_D}{12} \left( 3R_1^2 + 3R_2^2 + h^2 \right)$$

$$I_{Dy} = \frac{M_D}{2} \left( R_1^2 + R_2^2 \right)$$

$$S = \pi R_1^2$$

$$I = \frac{\pi R_1^2}{4}$$

The function \( f(y) \) that describes spatial coordinates of the orthogonal displacements \( u(y,t) = f(y)q_1(t) \) and \( w(y,t) = f(y)q_2(t) \) is derived by applying the
boundary conditions for this rotor bearing model given by classical Euler–Bernoulli first mode functions [11] for a non rotating free-free beam. The functions \( g(y), h(y) \) are the first and second derivatives respectively of \( f(y) \).

For an aligned journal bearing, classical 8 spring and damping coefficients are commonly used to model the dynamic radial force interactions between the journal and bearing. In this work 8 spring and damping coefficients are determined using the analytical expressions based on the short bearing approximation [8]. In Eqs. (5-6), the mass matrix is symmetric, while the damping and stiffness matrices are not symmetric. The non-symmetries arise from journal bearings and gyroscopic moments. If the net energy imparted to the rotor per cycle of harmonic motion by the non conservative force is negative, then the rotor bearing system is stable for the specified frequency, otherwise it is unstable [12].

**Stability Analysis**

The instability determination is based on the study of the rotor bearing system in free motion.

The characteristic equation for the rotor bearing system is:

\[
s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0
\]

where

\[
a_1 = \frac{(B_{11} + B_{22})}{m}; \quad a_2 = \frac{(K_{11} + K_{22}) + (B_{11}B_{22} - B_{12}B_{21})}{m^2};
\]

\[
a_3 = \frac{(B_{11}K_{22} + B_{22}K_{11} - B_{12}K_{21} - B_{21}K_{12})}{m^2}; \quad a_4 = \frac{(K_{11}K_{22} - K_{21}K_{12})}{m^2}
\]

**Unbalance Response**

The solution of equations of motion (Eqs. 5-6) with unbalance force of excitation is to be obtained in the form [5] as:

\[
q_1 = A_1 \sin \omega t + B_1 \cos \omega t
\]

\[
q_2 = A_2 \sin \omega t + B_2 \cos \omega t
\]

The substitution of Eqs. (8) – (9) in Eqs. (5) – (6), yields two equations to the coefficients of \( \sin \omega t, \cos \omega t \). The matrix forms of resulting linear set of equation are:

\[
\begin{bmatrix}
K_{11} - m\omega^2 & -B_{11}\omega & K_{12} & -B_{12}\omega \\
B_{11}\omega & K_{11} - m\omega^2 & B_{12}\omega & K_{12} \\
K_{21} & -B_{21}\omega & K_{22} - m\omega^2 & -B_{22}\omega \\
B_{21}\omega & K_{21} & B_{22}\omega & K_{22} - m\omega^2
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 \\
A_2 \\
B_2
\end{bmatrix}
= \begin{bmatrix}
m_s f(l_1)d_s\omega^2 \\
0 \\
0 \\
m_s f(l_1)d_s\omega^2
\end{bmatrix}
\]

\[
(10)
\]
The solution of Eq. (10) gives values of $A_1(\omega), B_1(\omega), A_2(\omega), B_2(\omega)$ which are functions of rotational speed.

The major and minor axis of the elliptical whirl orbits are [8]:

$$A, B = \sqrt{\left(\frac{A_1^2 + B_1^2 + A_2^2 + B_2^2}{2}\right)^2 \pm \left(\frac{\left(A_1^2 - B_1^2 + A_2^2 - B_2^2\right)^2 + 4\left(A_1B_1 + A_2B_2\right)^2}{2}\right)^2}$$

(11)

RESULTS AND DISCUSSION

The geometry of the rotor bearing system for the symmetric Kikuchi rotor is given in Table 1. The instability threshold speed for the disks-shaft-bearings configuration given in Table 1 is 880 rad/s. The roots of the characteristic equation (Eq. 7) are complex numbers [11] and the imaginary parts of the roots give the frequencies of the rotor bearing system. When the system becomes unstable, any one of the real part of complex roots become positive and this indicates growing whirl orbit. The influence of variation of journal bearing design variables on the instability threshold speed is given in Table 2. Bearing length, oil viscosity and radial clearance are chosen as the variables as the journal diameter and load on the bearing are influenced by the rotor design. Instability threshold speed for $L_b/D$ ratio of 0.5 ($L_b=0.02$ m) is higher compared to $L_b/D$ ratio of 1.0 and 1.5 ($L_b=0.04$ m and $L_b=0.06$ m). Reducing the $L_b/D$ ratio for a given load on bearing, increases the operating eccentricity ratio and hence increase in instability threshold. Decrease in oil viscosity from 0.06 Pas to 0.01 Pas also increases the operating eccentricity ratio and hence instability threshold speed. Instability threshold speed has decreased with increase in radial clearance from $150\times10^{-6}$ m to $250\times10^{-6}$ m. Increase in clearance obviously increases operating eccentricity ratio (as Sommerfeld number decreases), however increase in clearance also reduces the bearing spring and damping coefficients for a given load and speed condition. The influence of distance between the bearing supports on the instability threshold is given in Table 3. The flexural displacement of the beam for the end conditions (free-free) is zero when $l_o=0.238$ m, and hence choice of bearing supports at this location results in a lowest value of instability threshold speed (2.3 rad/s) which indicates that, the instability threshold is lowest when the flexural displacement of rotor are maximum.

Table 1 Geometry for the Rotor bearing system

<table>
<thead>
<tr>
<th>Disk</th>
<th>Shaft</th>
<th>Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.02 m</td>
<td>$L$</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.119 m</td>
<td>$\rho$</td>
</tr>
<tr>
<td>$H$</td>
<td>0.04 m</td>
<td>$E$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>7800 kg/m$^3$</td>
<td>$\omega$</td>
</tr>
</tbody>
</table>
Table 2 Influence of journal bearing on the instability threshold speeds

<table>
<thead>
<tr>
<th>$L_b$, m</th>
<th>Instability threshold speed, rad/s</th>
<th>$\mu$, Pas</th>
<th>Instability threshold speed, rad/s</th>
<th>$C_r$, m</th>
<th>Instability threshold speed, rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>1118.0</td>
<td>0.01</td>
<td>999.8</td>
<td>150x10^-6</td>
<td>880.0</td>
</tr>
<tr>
<td>0.04</td>
<td>880.0</td>
<td>0.06</td>
<td>880.0</td>
<td>250x10^-6</td>
<td>682.7</td>
</tr>
<tr>
<td>0.06</td>
<td>891.0</td>
<td>0.1</td>
<td>887.2</td>
<td>350x10^-6</td>
<td>744.0</td>
</tr>
</tbody>
</table>

Table 3 Influence of bearing support span on the instability threshold speeds

<table>
<thead>
<tr>
<th>$l_o$, m</th>
<th>Instability threshold speed, rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.131</td>
<td>880.0</td>
</tr>
<tr>
<td>0.200</td>
<td>294.4</td>
</tr>
<tr>
<td>0.238</td>
<td>2.3</td>
</tr>
<tr>
<td>0.300</td>
<td>430.0</td>
</tr>
</tbody>
</table>

Figure 2 shows the Campbell diagram for symmetric rotor. Complex conjugates roots are obtained when the system becomes unstable. Figure 3 shows the response due to unbalance excitation for the rotor bearing systems. The unbalance mass is considered for the first rotor (disk) located at 0.331 m from the origin. The major and minor axes of elliptical orbit (Eq. 11) are obtained solving [8] the system of linear equations (Eq. 10). The major axis of the elliptical orbit is indicated as the amplitude of response in Fig. 3. The unbalance response amplitude for the symmetric rotor increases in magnitude until the threshold speed of instability and the amplitude of response is nearly constant in the unstable regime. The unbalance response exhibits forward whirl mode in the range of rotor speed considered in this study.

CONCLUSION

Rayleigh – Ritz method is applied to the rotating shaft with thin rigid disks and flexible journal bearing supports. The rotor is analyzed by classical beam end conditions (free-free) in two orthogonal planes. Kinetic energy and strain energy of the rotor elements and virtual work of flexible bearing supports are calculated. Lagrange’s equations are used to derive dynamic equations of rotor motion. Based on the analysis of disk-shaft-bearing geometry for the symmetric rotor configuration, the following conclusions are obtained

1. Journal bearing design variables such as bearing length, oil viscosity and radial clearance influence the instability threshold. Decreasing the following variables independently can increase rotor instability threshold speed
   a. bearing length
   b. oil viscosity
   c. radial clearance.
2. Rotor flexural displacements significantly influence the instability threshold speed. The rotor exhibits minimum instability threshold, when the rotor flexure is maximum.

![Kikuchi rotor](image1)

**Fig. 2 Campbell diagram**

![Kikuchi rotor](image2)

**Fig. 3 Unbalance response**

**NOMENCLATURE**

- $C_r$: radial clearance (m)
- $D$: journal diameter (m)
- $d_u$: unbalance mass location in the disk (m)
- $E$: Modulus of elasticity (Pa)
- $g$: gravitational acceleration (m/s$^2$)
- $H$: thickness of the disk (m)
- $I$: second moment of area of circular shaft, (m$^4$)
- $I_{Dx}$, $I_{Dy}$, $I_{Dz}$: inertia of the disk in x, y, z directions (Fig. 1) (kg m$^2$)
- $L$: length of the shaft (m)
- $l_0$: location of the bearings (Fig. 1) (m)
- $l_1$: location of the first disk from the origin of $xyz$ coordinate system, (m)
- $L_b$: length of the bearing (m)
- $M_p$: mass of the rotor (kg)
- $m_u$: unbalance mass of the rotor (kg)
- $R_1$: inner radius of the disk (m)
- $R_2$: outer radius of the disk (m)
- $S$: area of shaft (m$^2$)
- $W$: load on the flexible bearing supports
$x, y, z$ coordinate system of reference shown in Fig. 1 (m)

$\mu$ viscosity of lubricant (Pa·s)

$\rho$ density of lubricant (kg/m$^3$)

$\omega$ angular velocity of shaft (rad/s)

REFERENCES


