

# Control Of Joint Of A Low Cost Robot

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## **Abstract:**

In order to develop suitable control procedure for controlling the robot arms a simplified low cost model of a robotic joint was constructed. The model consists of a rotor representing a robot link, which is driven by a low cost d.c. motor through a flexible belt. Two rotary potentiometers are used as angular displacement transducers- one of them being coupled with the rotor shaft by means of a small coupling. The signals obtained from the potentiometers are digitised. A digital computer calculates the final control signal by evaluating the difference between the input and the output signal and subsequently using an appropriate control law. The final control signal is converted into analog from that is fed to the motor via a current amplifier. In this control system, the main objective is to control and stabilize the rotor representing the robot link in the smallest possible of time with a locally available low cost motor. The command can be given either by a joystick or by computer program. The PID control law is used to calculate the final control signal through a suitable computer program written in a high level language. The model is studied under several input conditions.

## **Introduction:**

The control of robot joint to maintain accurate positioning is a challenging task even today. In almost all the robot joints digital control is applied due to several advantages. In this type of control a digital computer is essential as a control element to process a number of input signals to generate an appropriate control signal. In all physical systems, signals are in analog form, i.e., a continuous function of time, while the control computer handles data only in digital or in discrete form. These require signal discretization and A-D / D-A interfacing. Conventional servo controllers prove to be unsuitable for the control of robotic joints because of complicated dynamics involved. Stringent performance specifications for modern manipulators are usually achievable with various kinds of Software Servo Controllers (SSC)<sup>1-4</sup>. The values of the analog gain elements of an SSC can be changed at each sampling interval according to the command produced by supervisory software that monitors the system continuously.

A number of more sophisticated control techniques have also come up for effective control of manipulators. Resolved acceleration control<sup>5</sup>, computed torque method<sup>6-8</sup>, model-referenced adaptive control<sup>9</sup> are some of them. But each of them demands much more computational power

of the computer making the control system relatively expensive in comparison to SSC where a low cost microprocessor like INTEL8088 may be used for necessary computations.

Usually each joint of an electrically operated robot has either a stepper motor which move in fine steps or motor of type ac or dc, which moves continuously. Stepper motors together with their controller are relatively expensive and control of ac induction motor is quite complicated. The objective of this paper is to focus attention on the construction of a cost effective demonstration robot that uses low cost motors like those found in tape recorders for controlling the joints with reasonable accuracy. For this purpose a simplified model of a robotic joint was fabricated using a low cost dc motor and the model was subsequently tested under different control conditions including SSC.

The model consists of a rotor representing a robot link, which is driven by a low cost d.c. motor through a flexible belt, with a substantially high transmission ratio.

### Mathematical Model:

Electrical motor driven articulated manipulators are very common. Usually the motor shaft in such manipulator is positioned away from the joint axis towards the base of the robot and is connected to the joint by means of a belt, chain or gear drive. Thus a 2-DOF two-rotor torsional vibration system is formed (fig.1). The driving smaller rotor represents the motor armature, small pulley, motor bearings and other rotary elements. The other rotor represents the joint to be controlled.

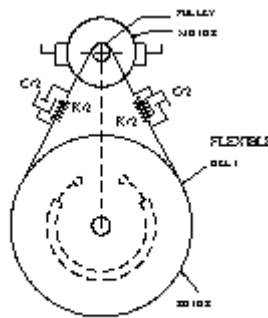


FIG:1 ROTOR UNIT

The mechanical torsional system is described by the set of two differential equations as given below:

$$J_m \ddot{\theta}_m + (C_m + C r_m^2) \dot{\theta}_m + K r_m^2 \theta_m - r_o r_m (C \dot{\theta}_o + K \theta_o) = T_m \quad (1)$$

$$J_r \ddot{\theta}_o + C r_o^2 \dot{\theta}_o + K r_o^2 \theta_o - r_o r_m (C \dot{\theta}_m + K \theta_m) = T_o \quad (2)$$

Where,

$J_m$  = moment of inertia acting at the motor shaft ( $\text{Kg-m}^2\text{-s}$ ).

$C_m$  = Coefficient of viscous damping at the motor shaft ( $\text{N-m-s/rad}$ ).

$T_m$  = Torque developed by the motor ( $\text{N-m}$ ),  $r_m$  = Radius of the motor pulley (m).

$J_r$  = moment of inertia of the driven rotor representing the robot joint ( $\text{Kg-m}^2\text{-s}$ ).

$C_r$  = Coefficient of viscous damping of the rotor ( $\text{N-m-s/rad}$ ).

$T_o$  = External torque acting at the joint ( $\text{N-m}$ ).

$r_o$  = Radius of the rotor pulley (m),  $K$  = Belt stiffness ( $\text{N-s/ m}^2$ ).

$C$  = Coefficient of damping of belt material ( $\text{N-s/m-rad}$ ).

$\theta_o$  = Angular displacement of the joint (rad).

$\theta_m$  = Angular displacement of the motor shaft (rad).

Motor torque  $T_m$  is obtained as,  $T_m = \mu \cdot I_a$

Where  $\mu$  and  $I_a$  are the motor torque constant and armature current of the motor respectively. The current amplifier circuit supplies the armature current and it is expressed as,

$$I_a = K_p' \cdot E_a$$

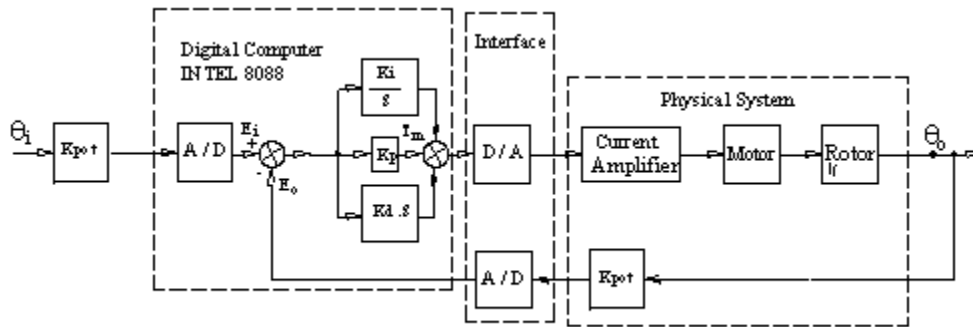


FIG.2 BLOCK DIAGRAM OF THE MODEL

Where  $K_p'$  is the gain of the current amplifier and  $E_a$  is derived from the error difference in voltage form using PID control law,

$$E_a = K_p \varepsilon + K_i \int \varepsilon dt + K_d d\varepsilon/dt \quad (3)$$

$$\text{Where } \varepsilon = K_{pot} (\theta_i - \theta_o) \quad (4)$$

Where  $\theta_i$  is the command generated either from a joystick or by a suitable computer program.

Thus the Motor torque becomes

$$T_m = \mu K_p' E_a \quad (5)$$

**Case Study:**

The effectiveness of the scheme is established with the construction of the model having following parameters:

$$J_m = 0.001 \text{ Kg-m}^2\text{-s}, C_m = 0.1 \text{ N-m-s/rad}, \mu = 0.5 \text{ N-m/amp}, r_m = 6 \text{ mm}, J_r = 1.5 \text{ Kg-m}^2\text{-s}.$$

$$C_r = 0.1 \text{ N-m-s/rad}, K = 500 \text{ KN-s/m}^2, C = 5 \text{ N-s/m-rad}, r_o = 50 \text{ mm} \text{ and } K_p = 50.$$

The authors tried to control and stabilize the rotor arm in the smallest possible time with minimum overshoot for different types of input. The command can be generated either manually from the joystick or from the computer program. The command from the joystick is in analog form and is digitized by the computer to calculate the error signal. The command from the computer is either an ideal step or a practical step simulated by developing a computer model of the physical system. First the authors optimized the values of system parameters of the computer model to get a response, which exactly follows the real model without any oscillation. The gain values like proportional gain ( $K_p$ ); differential gain ( $K_d$ ) and integral gain ( $K_i$ ) are optimized to get the minimum error for the stable system.

The response of the model was studied under these four different input conditions. In first case, ideal step input was realized and the rotor was allowed to follow the value of the given step. In next case, instead of generating an ideal step input, a practical step input was realized by forming a reduced order computer model of the system considering the system to be of second order. In the very next case, the computer model is considered to be of fourth order and the input is simulated from the computer model. In the last case, arbitrary command is given from the joystick and the model was allowed to track this joystick command.

**Simulation:**

Two simulation programs were used to calculate the optimum values of the proportional gain ( $K_p$ ), differential gain ( $K_d$ ) and integral gain ( $K_i$ ) for the minimum error of the system. In order to develop a computer model of the physical system, which is fabricated, the governing equations were first derived and then the system parameters were determined by experiment, which was subsequently verified by running a computer simulation program. PID control law was used to get the final control signal. Thus three gain values  $K_p$ ,  $K_d$  and  $K_i$  are required and they are adjusted to minimize the error. For that reason a simulation program is run to generate a particular combination for which the error becomes the minimum i.e., the response of the real system closely matches with the command at a minimum time.

**Implementation of Feedback:**

The control system used in this model follows PID control law. The command is either generated inside the control computer or it is taken into the computer after A-D conversion from the

joystick-operated potentiometer. On the other hand the output from the potentiometer connected to the rotor shaft after A-D conversion is also fed to the computer. The error is thus formed inside the control computer that acts as the feedback element. The time derivative and the time integral of the error are also computed inside the computer. A final control signal is then generated by adding the error, time integral of the error and time derivative of the error after multiplying the gains (  $K_p$  ,  $K_I$  ,  $K_d$  ) respectively to each of them.

Err (I) = proportional error signal = Input signal – output signal.

$$Derr = \text{differential error} = \frac{\text{Err}(I+h) - \text{Err}(I)}{h}$$

$$\begin{aligned} \text{And Ierr} &= \text{Integral error} \\ &= 0.5 * [\text{Err}(I+h) + \text{Err}(I)] * h \end{aligned}$$

Where, h is the sampling interval.

Err (I+h) = present error and Err (I) = last error before time h.

Then the final control signal is calculated as,

Control signal. =  $K_p * \text{Err} + K_I * \text{Ierr} + K_d * \text{Derr}$

The optimum values of the gains ( $K_p$ ,  $K_I$  ,  $K_d$  ) are obtained by suitable simulation. D-A converter is used with a sample and hold device to convert the digital signal to corresponding analog signal, which is then amplified by a current amplifier, and the amplified signal is used to run the dc motor.

### **Results and Discussion:**

The physical model was initially studied with the help of digital computer simulation. Initially an operational amplifier circuit was used as differential amplifier to generate the control signal by obtaining the error. It was found in that case that in order to minimize the error it was not possible to improve the gain value more than 10. The necessity to decrease the error led to the utilization of digital computer as an element in the control system. With the help of digital computer it is easily possible to implement the control laws with much higher gain values to obtain a desired system performance. The control system was developed and studied with some programs developed to control the rotor representing the robot arm. Firstly, a computer model of the physical system is developed with the optimum values of the system parameters and then that model is used to simulate the input signals for the real model. Then the optimum gain values are used to generate the final control signal, which is actually fed to the motor. It was found that for a combination of  $K_p = 50$ ,  $K_d = 6.5$ ,  $K_I = 0.3$ , the error becomes minimum. For the ideal step input command, it was noticed that even with a high gain value ( $K_p = 50$ ) the error is substantially high particularly at the initial stage. It is because of the fact that the

input is a pure step while the output is not at all a step one, rather a variable one and takes time to reach the input. In the next case, that is, in the case of practical input command, the above problem is taken care of by giving an input, which is simulated from a second order model framed inside the computer. Here the error is reduced but not so small which can be considered to be within the acceptable limit. It is because of the fact the actual system is of fourth order system and the computer model is of second order. The main problem in this case is that many variables are to be considered for manipulation whenever needed, which is cumbersome. The plots are taken directly and shown in fig3.

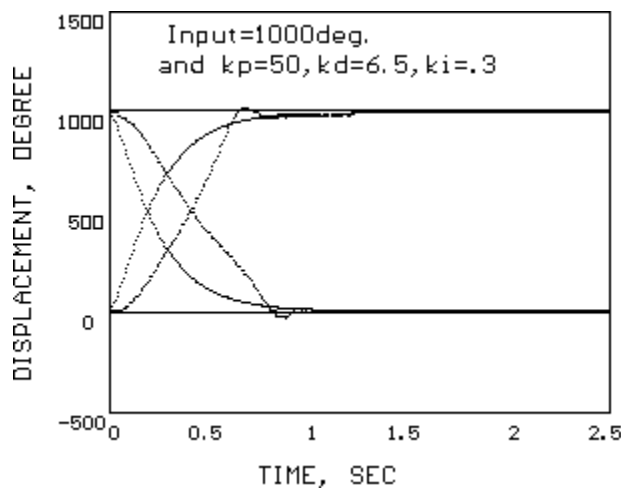


Fig.3 Response under Practical Input.

In the next case instead of an input simulated from a second order model, input is given from the formula as given below, which closely matches with the fourth order model.

$$\theta_i = \frac{t}{t_n} \frac{\sin \frac{2 * \pi * t}{t_n}}{2 * \pi} \text{ for } t \leq t_n$$

$$= 1 \text{ for } t > t_n \text{ where } t_n \text{ reach time to reach Input}$$

In this case just by controlling the reach time it is possible to control the error also and the plots are as shown in fig4. Here the problem of handling so many variables is totally eliminated. In both these two practical commands, it was observed that for very small input like for 50 degree or less the response becomes stepped as shown in fig5. This is because of the signal discretization error. In the next case, the problem of tracking is studied. Here the input is given manually through Joystick, which is then digitized and compared with the digitised position of the rotor generating the error signal.

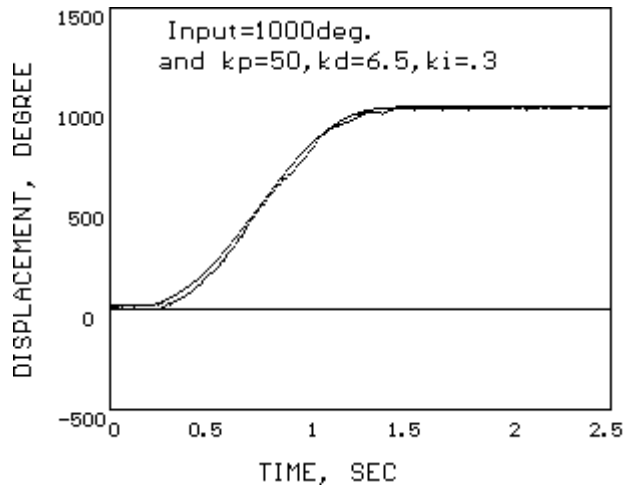


Fig.4 Response under more practical Input.

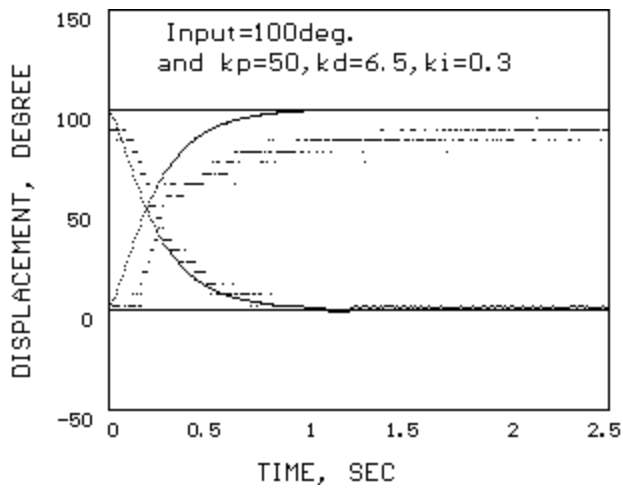


Fig.5 Response under small practical Input.

**Conclusions:**

By practical operation it was understood that the arbitrary position of the rotor is smooth and with respect to the input can be managed well by adopting one or all of the following

- Increased gain of the controller.
- Using a more powerful motor.
- Motor having more number of poles.

The above proposal will have to be replicated for each of the individual joint separately. In practice, the moment of inertia will depend on the link configuration and the payload of the robot. As the application is aimed at demonstration robots with small payloads, complicated dynamics of the link configuration like the coriolis component of acceleration and the centrifugal action is not considered.

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